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# The Unemployment Volatility Puzzle: The Role of Matching Costs Revisited

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## Abstract

Recently, Pissarides (2008) has argued that the standard search model with sunk fixed matching costs increases unemployment volatility without introducing an unrealistic wage response in new matches. We revise the role of matching costs and show that when these costs are not sunk and, therefore, can be partially passed on to new hired workers in the form of lower wages, the amplification mechanism of fixed matching costs is considerably reduced and wages in new hired positions become more sensitive to productivity shocks.

*Keywords:* unemployment volatility puzzle, search and matching, matching costs

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# 1 Introduction

The Mortensen-Pissarides (MP) search and matching model (Mortensen and Pissarides, 1994; Pissarides, 1985, 2000) studies the dynamics of unemployment in an environment where jobs are continuously created and destroyed. A sequence of papers by Costain and Reiter (2008), Hall (2005) and Shimer (2005) have questioned the model’s ability to match the U.S. data in one important dimension: the cyclical variations in unemployment in response to productivity shocks of reasonable size. For example, Shimer shows that under a reasonable calibration strategy, the MP model predicts that the vacancy-unemployment ratio and the average labor productivity should have nearly the same volatility. In contrast, the standard deviation of the vacancy-unemployment ratio in the U.S. is almost 20 times as large as the standard deviation of average labor productivity. This large discrepancy between the volatility implied by the model and the data constitutes an empirical puzzle, known as the unemployment volatility puzzle.

Pissarides (2008) shows that introducing fixed matching costs into the model (e.g., training costs) can significantly increase the volatility of labor-market outcomes, such as tightness and the job finding rate. He points out that this result is obtained without inducing a counterfactually low volatility in the wages of new jobs. In his quantitative exercise, Pissarides only considers *sunk* fixed matching costs. That is, “they are sunk once the wage bargain is concluded and the worker takes up the position”. He shows that when these costs increase from zero to 40 percent of average labor productivity, the volatility of the vacancies-unemployment ratio (measured by its elasticity) increases almost twofold, and it matches the observed volatility in the U.S. labor market. He also argues that non-sunk fixed training costs play a similar role.

In this paper we evaluate the amplification mechanism of *non-sunk* fixed matching costs, and examine whether the cyclical volatility is substantially

augmented. We show that when these costs are not sunk and, therefore, can be partially passed on to workers through lower wages, the volatility of the vacancy-unemployment ratio is approximately an order of magnitude less responsive to variations in these costs. Thus, from a quantitative standpoint, the contribution of fixed matching costs in explaining labor market volatility depends not only on the level, but also on what proportion of these costs is sunk. Moreover, we observe that *non-sunk* fixed matching costs may also introduce a significant change in the volatility of wages of new hired workers.

## 2 The model

Given that our model is essentially the same as Pissarides' (2008), its presentation is reduced to a minimum. In this economy, there is a continuum of risk-neutral, infinitely-lived workers and firms which discounts future payoffs at a common rate  $r$ ; capital markets are perfect; and time is continuous.

There is a time-consuming and costly process of matching workers and job vacancies, captured by a standard constant-returns-to-scale matching function  $m(u, v) = m_o u^\eta v^{1-\eta}$ , where  $u$  denotes the unemployment rate,  $v$  is the vacancy rate, and  $\eta$  and  $m_o$  are the function parameters. Unemployed workers find jobs at the rate  $f(\theta) = m(u, v)/u$ , and vacancies are filled at the rate  $q(\theta) = m(u, v)/v$ , where  $\theta = v/u$  denotes labor market tightness. From the properties of the matching function, the higher the number of vacancies with respect to the number of unemployed workers, the easier it is to find a job,  $f'(\theta) > 0$ , and the more difficult it is to fill up vacancies,  $q'(\theta) < 0$ .

A job can be either filled or vacant. Before a position is filled, the firm has to open a job vacancy with a flow cost  $c$ . Firms have a linear technology with labor as the only production factor. Each filled job yields instantaneous profit equal to the difference between labor productivity  $p$  and the wage. When the

worker arrives, the firm pays fixed costs  $H$  which is sunk. Moreover, it pays non-sunk fixed costs  $T$  right after both the firm and the worker agree to start a working relationship. A job remains “new” until a shock with arrival rate  $\lambda$  hits the match and changes its status to a continuing job. In that case, the worker and the firm renegotiate wages. Notice that  $T$  becomes sunk after the initial negotiation. Therefore, new and continuing jobs will have different wages  $w^n$  and  $w^c$ , respectively. Thus, the value of vacancies  $V$ , the value of a new job  $J^n$ , and the value of a continuing job  $J^c$  are represented by the following Bellman equations:

$$rV = -c + q(\theta)(J - H - T - V), \quad (1)$$

$$rJ^n = p - w^n + s(V - J^n) + \lambda(J^c - J^n), \quad (2)$$

$$rJ^c = p - w^c + s(V - J^c), \quad (3)$$

When finding a job, the unemployed worker first belongs to a new job. At rate  $\lambda$ , it becomes a continuing job. All employed workers separate from their firm at the constant rate  $s$ . Unemployed and employed workers’ Bellman equations are given by

$$rU = z + f(\theta)(W^n - U), \quad (4)$$

$$rW^n = w^n + s(U - W^n) + \lambda(W^c - W^n), \quad (5)$$

$$rW^c = w^c + s(U - W^c), \quad (6)$$

where  $z$  represents the flow utility from leisure.

As is standard, we assume that there is free entry for vacancies. Therefore, in equilibrium:

$$V = 0. \quad (7)$$

We also assume that wages in new jobs are determined through bilateral Nash bargaining between the worker and the firm. The first-order conditions

for entrant employees yield the following equation:

$$(1 - \beta)(W^n - U) = \beta(J^n - T), \quad (8)$$

where  $\beta \in (0, 1)$  denotes the workers' bargaining power relative to firms'. Note that the Nash condition depends on matching costs  $T$  but not  $H$  because the former are not sunk to new jobs, and therefore they are explicitly considered in the wage negotiation with new entrants.

This sharing rule implies that  $J^n - T = (1 - \beta)S^n$ , where  $S^n = J^n + W^n - U - T$  is the surplus of a new job (net of sunk cost  $H$ ). Using all the value functions (1)-(6) and the zero-profit condition (7), we obtain the equilibrium job creation condition

$$\frac{(1 - \beta)(p - z) - \beta(c\theta + f(\theta)H)}{r + s} = \frac{c}{q(\theta)} + H + (1 - \beta)T. \quad (9)$$

As Pissarides (2008) points out, this job creation condition is independent of the specific wage determination scheme for continuing jobs. If, in particular, we assume a Nash wage rule for continuing matches as well, we obtain the following equilibrium wages:

$$w^n = (1 - \beta)z + \beta(c\theta + p + f(\theta)H - (r + s + \lambda)T), \quad (10)$$

$$w^c = (1 - \beta)z + \beta(c\theta + p + f(\theta)H). \quad (11)$$

Since  $H$  are sunk, they increase the implicit bargaining power of all workers and, therefore, their wages. In contrast, firms can pass on part of the non-sunk matching costs  $T$  to new employees in the form of lower wages.

A steady-state equilibrium in this economy is a triplet of labor market tightness and wage rates  $(\theta^*, w^{n*}, w^{c*})$  that solves equations (9), (10), and (11) for the steady-state productivity level  $p^*$ .

### 3 Parameter values and elasticities

For comparative purposes, we use the same targets and parameter values as in Pissarides (2008), and calibrate the model at monthly frequency without fixed matching costs,  $T = H = 0$ . Without additional information, we assume an average duration of one quarter before new hired jobs are converted to continuing jobs (i.e.,  $1/\lambda = 3$ ). Notice that the arrival job conversion rate  $\lambda$  becomes irrelevant when  $T = 0$ . See Table 1 for more details. Then, we increase either the sunk ( $H$ ) or non-sunk ( $T$ ) matching costs and adjust the vacancy parameters  $c$  in order to maintain the same steady-state value for the labor market tightness  $\theta^*$  and, therefore, the equilibrium unemployment rate  $u^* = \frac{s}{s+f(\theta^*)}$ .

The central question in this paper is whether this extended MP matching model with fixed matching costs can explain the size of the business cycle fluctuations in labor-market tightness and unemployment given the separation rate. To explore this issue, we find the elasticities of the vacancy-unemployment ratio,  $\varepsilon_\theta$ , and wages in new jobs,  $\varepsilon_w$ , with respect to labor productivity  $p$ . Thus, from the job creation condition (9) and the wage equations (10), we obtain

$$\varepsilon_\theta = \frac{1}{\eta} \left[ \frac{(1-\beta)p^*}{(1-\beta)(p^* - z) + \beta \frac{1-\eta}{\eta} c\theta^* - [r + s + \beta \frac{1-2\eta}{\eta} f(\theta^*)]H - (r+s)(1-\beta)T} \right], \quad (12)$$

and

$$\varepsilon_w = \beta \left[ \frac{p^* + \varepsilon_\theta (c\theta^* + f(\theta^*)(1-\eta)H)}{w^{n^*}} \right]. \quad (13)$$

Table 2 shows these elasticities for different values of  $H$ ,  $T$  and  $\lambda$ . We find that the volatility of the vacancies-unemployment ratio  $\theta$  is much higher when sunk fixed matching costs  $H$  are increased. For example, the elasticity of the vacancies-unemployment ratio is multiplied almost by two (from 3.67 to 7.24) when these costs increase from 0 to 40 percent of the average labor

productivity. In contrast, this elasticity increases only by 5.72 percent (from 3.67 to 3.88) for the same variation in the non-sunk matching costs  $T$ . Thus, from a quantitative point of view, the amplification effect of fixed matching costs on labor market volatility depends not only on the level but also on what proportion of these costs is sunk.

To understand this result, notice that there are two effects. There is a direct effect associated with the terms that depend on  $H$  and  $T$  in the denominator of (12). It is easy to see that if  $\eta > 1/(r + s + 2)$ , as in our parametrization, then  $r + s + \beta \frac{1-2\eta}{\eta} f(\theta^*) > (r + s)(1 - \beta)$  and, consequently, an increase in  $H$  has a larger positive impact on  $\varepsilon_\theta$ . Furthermore, we have an indirect effect through the recalibration of parameter  $c$  as explained above. Note that an increase in  $H$  causes  $\theta^*$  to fall more compared to the impact of  $T$ . Therefore, in order to keep  $\theta^*$  constant,  $c$  has to fall more when  $H$  increases. Clearly,  $\varepsilon_\theta$  is decreasing in  $c$ . Thus, the indirect effect of a change in fixed matching costs on  $\varepsilon_\theta$  through  $c$  is larger for  $H$ . Provided that  $\eta > 1/(r + s + 2)$ , both effects are bigger in the case of a change in  $H$ , which explains why  $\varepsilon_\theta$  increases more when we raise  $H$ .

The question now is to what extent each effect contributes to this result. Given our parametrization we find that for an increase in fixed matching costs (either  $H$  or  $T$ ) from 0 to 0.1, the direct effect explains about 23 percent of the difference in the variation of  $\varepsilon_\theta$ . When these costs go up to 0.4, the direct effect accounts for about 13 percent of the difference. Therefore, it seems that the large impact that sunk costs have on  $\varepsilon_\theta$  is mainly due to the indirect effect through the recalibration of  $c$ .

Finally, notice that for  $\lambda$  near zero both  $H$  and  $T$  do not introduce significant changes in the elasticity of wages in new matches. It remains near one in both cases. However, when the arrival job conversion rate increased from zero to one the elasticity of new hired wages,  $\varepsilon_{w^n}$ , jumps from 1.00 to



1.26 when  $T = 0.40$ . Thus, fixed non-sunk matching costs may violate the near-proportionality between wages in new matches and labor productivity estimated in Haefke, Sonntag, and van Rens (2007) as well as in Pissarides (2008).

For example, under the assumption that  $H$  and  $T$  only capture training costs, we find that this source of labor turnover costs is able to match the unemployment volatility if nearly 20 percent of them are sunk. More specifically, in order to examine the relevance of training costs in the U.S. labor market, we use information reported by Barron, Berger and Black (1997) that comes from the 1982 Employer Opportunity Pilot Project, a cross-sectional firms-level survey containing detailed information on these labor turnover costs. According to the authors, 95 percent of new hired workers received some kind of training and spent, on average, 142 hours in training activities during the first quarter in the firm.<sup>1</sup> When adding the contribution of incumbent workers and supervisors in training new employees, which is placed at 87.5 hours on average, the resulting cost amounts to 66 percent of the quarterly wage of a new hire.<sup>2</sup> Thus, as is shown in the last row of Table 2, with  $1/\lambda = 3$ ,  $H = 0.285$ , and  $T = 1.185$ , which implies  $w^{n*} = 0.7427$ , the model is able to match the observed U.S. unemployment volatility of 7.56 calculated by Pissarides (2008). However, under this scenario, wages in new matches are about 33 percent more sensitive to labor productivity shocks than in the data.

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<sup>1</sup>Using a more recent survey, the 1992 Small Business Administration survey, Barron, Berger and Black (1997) report a similar number of hours spent on on-the-job training during the first three months of employment (150 hours).

<sup>2</sup>For more information, see Table 1 in Silva and Toledo (2009).

## 4 Conclusion

In a recent paper, Pissarides (2008) argues that the presence of fixed matching costs can improve the volatility of unemployment maintaining the one-to-one response of wages to productivity fluctuations observed in the data. In his model, the matching costs are sunk, so new matched workers take actions designed to extract the quasi-rents created by them. We show that when the fixed matching costs can be partially passed on to workers through lower wages, the volatility of the vacancy-unemployment ratio is significantly reduced. Moreover, we also observe that *non-sunk* fixed matching costs introduce changes in the elasticity of wages of new hired workers and may violate its proportionality with respect to labor productivity shocks.

Finally, although there are important quantitative differences related to the impact of sunk and non sunk fixed matching costs on unemployment volatility, these type of costs can be considered empirically relevant and still help to improve the amplification mechanisms of the matching model.

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Table 1: Calibrated parameter values for the U.S. economy

Parameter	Value	Source/Target
Labor productivity, $p^*$	1	Normalization
Exogenous separation probability, $s$	0.036	Data (Shimer, 2005)
interest rate, $r$	0.004	Data
Employment opportunity cost, $z$	0.677	Hall & Milgrom (2008)
Matching function elasticity, $\eta$	0.5	Petrongolo & Pissarides (2001)
Matching function scale, $m_o$	0.7	To match the job finding prob.
Workers' bargaining power, $\beta$	0.5	$\beta = \eta$ (efficiency)
Cost of vacancy, $c$	0.034	Solves (9)
Sunk fixed matching costs, $H$	0	Benchmark
Non sunk fixed matching costs, $T$	0	Benchmark
Arrival job conversion rate, $\lambda$	0.333	Benchmark
Variable		
Labor market tightness, $\theta^*$	0.72	JOLTS
Job finding probability, $f(\theta^*)$	0.594	Shimer (2005)

Table 2: Short-run effects of sunk vs. non-sunk fixed matching costs

$\lambda$	$H$	$T$	$c$	$\varepsilon_\theta$	$\varepsilon_w^n$
0.33	0.00	0	0.356	3.666	0.985
0.33	0.10	0	0.273	4.182	0.989
0.33	0.20	0	0.191	4.867	0.995
0.33	0.30	0	0.108	5.821	1.000
0.33	0.40	0	0.026	7.238	1.013
0.33	0	0.00	0.356	3.666	0.985
0.33	0	0.10	0.351	3.717	1.006
0.33	0	0.20	0.346	3.770	1.028
0.33	0	0.30	0.343	3.824	1.050
0.33	0	0.40	0.336	3.880	1.074
0	0	0.40	0.336	3.880	1.000
0.25	0	0.40	0.336	3.880	1.055
0.50	0	0.40	0.336	3.880	1.116
0.75	0	0.40	0.336	3.880	1.184
1.00	0	0.40	0.336	3.880	1.261
0.33	0.285	1.185	0.062	7.560	1.331